MATH 33A lecture 2, Linear Algebra

Sample Final Exam

Instructor Kostadinov

NAME (please print legibly): _____

Your University ID Number: _____

- **No calculators, notes, friends or other people taking the exam for you are allowed.**
- **Please show all your work. You may use back pages if necessary.**

Part A		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
TOTAL	120	

Part A

1. (20 points) Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$

1. Find all eigenvalues.
2. Find eigenvectors for the corresponding eigenvalues and show that they form a basis of $\mathbb{R}^2$.
3. Find the matrix of A with respect to this basis of eigenvectors.
4. Find A^6 with the help of part (3) above.

2. (20 points) Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$

1. Show that the only distinct eigenvalues of A are 3 and 5.

2. Find a basis for the eigenspace $E_3 = Ker(A - 3I)$ and a basis for the eigenspace $E_5 = Ker(A - 5I)$ and show that the dimensions of the eigenspaces add up to 3. This means that A is diagonalizable, i.e. there is a basis of eigenvectors for $\mathbb{R}^3$ with respect to which the matrix of A is diagonal. Find this diagonal matrix that represents A in this new basis.

3. (20 points) Find the least-squares solution $\vec{x}^*$ of the following inconsistent system

$$A\vec{x} = \vec{b}, \quad \text{where} \quad A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

4. (20 points) Consider the subspace W of $\mathbb{R}^4$, spanned by the vectors

$$\vec{v_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{v_2} = \begin{pmatrix} 1 \\ 9 \\ -5 \\ 3 \end{pmatrix}$$

Find the matrix of the orthogonal projection onto W. Explain why the matrix of any orthogonal projection cannot be an orthogonal matrix.

5. (20 points) Is it always true that

$$\text{rank}(A) = \text{rank}(A^T A)$$

for all $n \times m$ matrices A? Explain.

6. (20 points) Work on all problems given previously on both sample and actual exams.